E-Z Sequence

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***Abstract*—I have derived a sequence, the E-Z Sequence, which finds the highest value of m such that $z^m|n$ where $z \geq 2$ and $m, n, z \in \mathbb{N}$ using the recursive sequence of**

$$a_n = \left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}$$
$$\text{where } a_1 = \left\{\{0\}^{z-1}\right\}$$

such that the value of m can be found in the sequence where n is the n^{th} position in the sequence and m is the corresponding number. In this paper, I will describe the derivation of the E-Z Sequence. In addition to this, I will also provide a proof of correctness for the E-Z Sequence and a proof that the length of the E-Z Sequence at any value of n for a_n is $z^n - 1$. Applications of the E-Z Sequence include to calculate primes. When the E-Z Sequence is compared to itself at every value of z, we are able to find every natural divisor of every natural number. Therefore, if there exists a p^{th} position in the E-Z Sequence such that the corresponding number is 0 for every z except for in the case where the sequence which indicates the value of m for $p^m|p$ as 1, then this number is prime because it has no natural divisors greater than 1 other than itself.

I. Introduction

In my previous paper, I suggested that a "possibility for future work is to determine primes using the Binary Carry sequence and sequences like it. The Binary Carry sequence is used to compute the value of $m \in \mathbb{N}$ for every $n \in \mathbb{N}$ such that $2^m|n$ where n is the n^{th} position in the sequence and m is the corresponding number. The first step to computing primes using this approach is to discover sequences that indicate the value of $m \in \mathbb{N}$ for every $n \in \mathbb{N}$ such that $3^m|n$, $4^m|n$, $5^m|n$, and so on. The next step would be to create a Turing machine to compute the values of each position in the sequence for every sequence we have discovered. Then, for the i^{th} position in any sequence, a Turing machine exists to calculate the sequence that indicates the value of m for $i^m|n$. When all the Turing machines are ran simultaneously, we are able to observe every natural divisor of every natural number. For any i^{th} position in any sequence, say we observe that every corresponding m value is 0 except for in the sequence which indicates the value of m for $i^m|n$ such that this value is 1. Hence, the number i has only the divisor of itself and 1, therefore i is prime." [1]

In this paper, I will describe a sequence that indicates the highest value of $m \in \mathbb{N}$ for every $n \in \mathbb{N}$ such that $z^m|n$ where $z \in \mathbb{N}$ and $z \geq 2$. This is the first step to the process of discovering primes which I described.

A. Binary Carry Sequence

For the purpose of understanding exactly what the sequence indicating the highest value of $m \in \mathbb{N}$ for every $n \in \mathbb{N}$ such that $z^m|n$ where $z \in \mathbb{N}$ and $z \geq 2$ accomplishes, I will give a description and explanation of the sequence when $z = 2$. This sequence is called the Binary Carry Sequence and has been discovered to correlate with other mathematical problems. Some of these correlating problems are described in my previous paper if more information is desired.

"Let $z \in \mathbb{N}$. Then for every natural number n, there exists a natural number m such that z to the power of m evenly divides n. There exists such a sequence, the Binary Carry sequence, that will indicate the value of m for every n for the value of $z = 2$. The Binary Carry sequence can be derived using the recursive equation $a_n = \{a_{n-1}, n-1, a_{n-1}\}$ where $n \in \mathbb{N}$ and $a_1 = \{0\}$. The first few derivations of the sequence are shown in the table below." [1]

n	a_n
1	0
2	0 1 0
3	0 1 0 2 0 1 0
4	0 1 0 2 0 1 0 3 0 1 0 2 0 1 0
5	0 1 0 2 0 1 0 3 0 1 0 2 0 1 0 4 0 1 0 2 0 1 0 3 0 1 0 2 0 1 0

"The value of m for every n such that $2^m|n$ can be found in the sequence where n is the n^{th} position in the sequence and m is the corresponding number. For example, to find the value of m for 8 where $2^m|8$, we will locate the 8^{th} position in the sequence of which the corresponding value is 3. Therefore, the highest power of 2 which evenly divides 8 is 3, hence $2^3|8$." [1]

B. Applications

Application for this sequence which has been previously mentioned is to discover prime numbers. The sequence discovers primes in such a manner that multiple sequences must be used simultaneously. The sequence described in this paper, the E-Z Sequence, indicates the highest value of $m \in \mathbb{N}$ for every $n \in \mathbb{N}$ such that $z^m|n$ where $z \in \mathbb{N}$ and $z \geq 2$. When the E-Z Sequence is compared to itself at every value of z, we are able to indicate every natural divisor of every natural number [1]. Therefore, if there exists a p^{th} position in the E-Z Sequence such that the corresponding number is 0 for every z except for in the case where the sequence which indicates the value of m for $p^m|p$ as 1, then this number is prime because it has no natural divisors greater than 1 other than itself [1].

Future research on the topic can be to find where the E-Z Sequence is applicable.

II. Approach

A. Notation

For the purposes of this paper, I have created a legend clarifying the meaning of each notation used. For this table, let $z, m, n, p \in \mathbb{N}$ and a_{n-1} represent a sequence.

$Notation$	$Meaning$
z^m	z to the exponential power of m
$z \mid n$	z evenly divides n
$\{\{0\}^p\}$	$\{0, 0, ..., 0\}$ where 0 is repeated in the sequence p-times
$\{\{n-1, a_{n-1}\}^p\}$	$\{n-1, a_{n-1}, n-1, a_{n-1}, ..., n-1, a_{n-1}\}$ where $n-1, a_{n-1}$ is repeated in the sequence p-times
$\lvert a_n \rvert$	the number of elements in the sequence a_n

This notation will be used throughout the paper, so a close study of this table will be beneficial before continuing.

B. Determining the E-Z Sequence

In order to find the sequence which indicates the highest value of $m \in \mathbb{N}$ for every $n \in \mathbb{N}$ such that $z^m|n$ where $z \in \mathbb{N}$ and $z \geq 2$, I started by observing the sequences for $z = 2, 3, 4....$ I created a table describing each sequence of numbers such that the n^{th} position in the sequence indicates the highest value of z such that $z^m|n$. Below an example of the table I created to easily observe the sequences.

z	highest value of z such that $z^m \mid n$ for the value of the n^{th} position
2	0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,4,0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,5,0,1,0,2,...
3	0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,3,0,0,1,0,0,1,0,0,2,...
4	0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,2,0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,2,0,0,0,1,...
5	0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,2,0,0,0,0,1,0,0,0,0,1,0,...
6	0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,2,...

When observing these sequences, I compared them all to the Binary Carry Sequence $a_n = \{a_{n-1}, n-1, a_{n-1}\}$ where $n \in \mathbb{N}$ and $a_1 = \{0\}$. I noticed that $a_1 = \left\{\{0\}^{z-1}\right\}$ for each sequence. Then, when observing how often each other number occurs, I saw that 1 will occur between a_1 $z-1$ times before a different number, 2, occurs. This sequence before 2 occurs is denoted a_2. Then, 2 will occur between a_2 $z-1$ times before a different number, 3, occurs. This sequence before 3 occurs is denoted a_3. This pattern continues for every number $n \in \mathbb{N}$. Using this information, I derived the following sequence, which I will refer to as the E-Z sequence.

$$a_n = \left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}$$
$$\text{where } a_1 = \left\{\{0\}^{z-1}\right\}$$

This sequence finds the highest value of m such that $z^m|n$ where $z \geq 2$ and $m, n, z \in \mathbb{N}$.

C. Examples of the E-Z Sequence

For a thorough understanding of how the E-Z sequence operates, I will give examples of the sequence for a few small numbers. The E-Z Sequence operates the same for all values of $z \in \mathbb{N}$ such that $z \geq 2$.

For $z = 2$, the highest power of m such that $2^m|n$ can be found using the sequence

$$a_n = \{a_{n-1}, n-1, a_{n-1}\}$$
$$\text{where } n \in \mathbb{N} \text{ and } a_1 = \{0\}$$

such that n is the n^{th} position in the sequence and m is the corresponding number. This is derived from the E-Z Sequence such that $\{n-1, a_{n-1}\}$ occurs $2-1 = 1$ time in the sequence a_n and $\{0\}$ occurs $2-1 = 1$ time in the sequence a_1.

For $z = 3$, the highest power of m such that $3^m|n$ can be found using the sequence

$$a_n = \{a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}\}$$
$$\text{where } n \in \mathbb{N} \text{ and } a_1 = \{0, 0\}$$

such that n is the n^{th} position in the sequence and m is the corresponding number. This is derived from the E-Z Sequence such that $\{n-1, a_{n-1}\}$ occurs $3-1 = 2$ times in the sequence a_n and $\{0\}$ occurs $3-1 = 2$ times in the sequence a_1.

For $z = 5$, the highest power of m such that $5^m|n$ can be found using the sequence

$$a_n = \{a_{n-1}, n-1, a_{n-1}, n-1,$$
$$a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}\}$$
$$\text{where } n \in \mathbb{N} \text{ and } a_1 = \{0, 0, 0, 0\}$$

such that n is the n^{th} position in the sequence and m is the corresponding number. This is derived from the E-Z Sequence such that $\{n-1, a_{n-1}\}$ occurs $5-1 = 4$ times in the sequence a_n and $\{0\}$ occurs $5-1 = 4$ times in the sequence a_1.

For $z = 9$, the highest power of m such that $9^m|n$ can be found using the sequence

$$a_n = \{a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}, n-1,$$
$$a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}, n-1, a_{n-1}\}$$
$$\text{where } n \in \mathbb{N} \text{ and } a_1 = \{0, 0, 0, 0, 0, 0, 0, 0\}$$

such that n is the n^{th} position in the sequence and m is the corresponding number. This is derived from the E-Z Sequence such that $\{n-1, a_{n-1}\}$ occurs $9-1 = 8$ times in the sequence a_n and $\{0\}$ occurs $9-1 = 8$ times in the sequence a_1.

D. Proof of Size of the E-Z Sequence

To find the size of the E-Z Sequence at any value of n for a_n, I calculated the size of a_n at any value of $n \geq 1 \in \mathbb{N}$ for any value of $z \geq 2 \in \mathbb{N}$. I began with finding $|a_1| = \left|\left\{\{0\}^{z-1}\right\}\right|$ and the number of times that 0 appears in the sequence is $z-1$, hence $\left|\left\{\{0\}^{z-1}\right\}\right| = z-1$. Then I found $|a_n| = \left|\left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}\right|$. Since $\{n-1, a_{n-1}\}^{z-1}$ repeats $\{n-1, a_{n-1}\}$ $z-1$ times, this implies that $\left|\{n-1, a_{n-1}\}^{z-1}\right| = (|n-1| + |a_{n-1}|)(z-1)$ and since $n-1$ is simply one element, $|n-1| = 1$. Hence, $\left|\{n-1, a_{n-1}\}^{z-1}\right| = (1 + |a_{n-1}|)(z-1)$. Then I calculated the value of $|a_n| = \left|\left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}\right|$.

$$\begin{aligned}
|a_n| &= \left|\left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}\right| \\
&= |a_{n-1}| + (|n-1| + |a_{n-1}|)(z-1) \\
&= |a_{n-1}| + (1 + |a_{n-1}|)(z-1) \\
&= |a_{n-1}| + z + z(|a_{n-1}|) - 1 - |a_{n-1}| \\
&= z + z(|a_{n-1}|) - 1 \\
&= z(|a_{n-1}| + 1) - 1
\end{aligned}$$

Note that

$$|a_n| = z(|a_{n-1}|+1)-1 \implies |a_{n-p}| = z(|a_{n-p-1}|+1)-1$$
$$\implies |a_{n-p}|+1 = z(|a_{n-p-1}|+1)$$

and $|a_n| = |a_1| \implies p = n-1$. So, if $|a_{n-p}|$ for $1 \geq p \geq n-2$ is continuously plugged into the equation derived for $|a_n|$, the following is derived.

$$\begin{aligned}|a_n| &= z\left(\left(\prod_{p=1}^{p=n-2}(z)\right)(|a_1|+1)\right)-1\\ &= z\left(\left(\prod_{p=1}^{p=n-2}(z)\right)(z-1+1)\right)-1\\ &= z\left(\left(\prod_{p=1}^{p=n-2}(z)\right)(z)\right)-1\\ &= z(z^{n-2}(z))-1\\ &= z^n-1\end{aligned}$$

Hence, the length of the E-Z Sequence at any value of n for a_n is $z^n - 1$.

E. Proof of Correctness of the E-Z Sequence

I will prove that the E-Z Sequence $a_n = \left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}$ finds the highest value of m such that $z^m|n$ where $z \geq 2$ and $m, n, z \in \mathbb{N}$ using the method such that the value of m for every n such that $z^m|n$ can be found in the sequence where n is the n^{th} position in the sequence and m is the corresponding number.

I will first prove that $a_1 = \left\{\{0\}^{z-1}\right\}$ holds for every value of z for every h^{th} position in the sequence where $h \in \mathbb{N}$. All of the elements in the sequence a_1 are 0, and there are $|a_1| = z-1$ elements in the sequence. Hence, for any $1 \leq h > z-1$, the sequence indicates that the highest power of z which evenly divides h is 0. Obviously, this is true.

Now I will prove that for any time when the sequence a_1 is repeated in the sequence, it will hold for every value of z for every h^{th} position in the sequence where $h \in \mathbb{N}$. Note that the only time 0 will occur in the sequence is when it is in the sequence a_1. When a_1 is repeated in the sequence $a_n = \left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}$, it is always separated from the next sequence of a_1 by exactly one number, $n-1$. This implies that the h^{th} positions corresponding to the values where a_1 is in the sequence are in the interval $xz+1 \leq h \leq xz+z-1$, which includes all z-1 positions which will correspond to the numbers in the sequence a_n only where the sequence a_1 occurs. Hence, the highest power of the h^{th} position from $xz+1 \leq h \leq xz+z-1$ such that $z^m|h$ will be $m=0$, which is clearly true.

Next I will prove that the sequence $a_n = \left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}$ will correctly indicate highest value of m such that $z^m|n$ where n is the h^{th} position in the sequence. I will show this by proving that the sequence will correctly compute the desired result for any sequence beyond a_1. So, I can derive that for any $n \in \mathbb{N}$, $a_1 + n = \left\{a_n, \{n, a_n\}^{z-1}\right\}$. It can be seen that n will occur after any occurrence of the sequence a_n in a_{1+n}. This implies that if $1 \geq x > z$ and $x \in \mathbb{N}$, n is the $x(|a_n|+1) = x(z^n-1+1) = xz^n$ number in the sequence. When n is the corresponding number in the sequence to position h, h is at position $x(z^n)$. Since $x \neq z$, the highest power that evenly divides h is n hence $z^n|qz^n$, which is obviously true.

Since I have proven that the sequence produced the desired results for a_1 and a_1+n, I can conclude that the sequence will produce the desired result for any value of n for a_n. Hence the E-Z sequence correctly finds the highest value of m such that $z^m|n$ such that n is the n^{th} position in the sequence and m is the corresponding number for every sequence of a_n for any z.

III. Conclusion

In conclusion, the E-Z Sequence finds the highest value of m such that $z^m|n$ where $z \geq 2$ and $m, n, z \in \mathbb{N}$ using the recursive sequence of

$$a_n = \left\{a_{n-1}, \{n-1, a_{n-1}\}^{z-1}\right\}$$
$$\text{where } a_1 = \left\{\{0\}^{z-1}\right\}$$

such that the value of m can be found in the sequence where n is the n^{th} position in the sequence and m is the corresponding number.

I have proven that the length of the E-Z Sequence for any value of z at a_n is z^n-1 and that the E-Z Sequence will correctly indicate the value of m for every n under every sequence of $z \geq 2$.

As a result of proving that the E-Z Sequence exists and that is is correct, I have also proven that there exists a sequence which exists for every natural number $z \geq 2$ such that for every natural number n, the natural number m such that z to the power of m evenly divides n is indicated.

Future work on the topic includes implementing the E-Z Sequence to find prime numbers. Because the method had been created, the only challenge now is to find an efficient way to implement the method described in the Application section. I am confident that writing code to calculate and compare the E-Z Sequences will not be too difficult of a problem. Because the run time of implementing this algorithm is unknown at the moment, the efficiency of this problem is also unknown. Future work can also be to calculate the run time of this algorithm to finding primes and seeing if the run time is less than algorithms that currently exist for finding primes.

I am hopeful that this method will be researched more thoroughly and implemented such that more primes will be discovered.

References

[1] K. Brown, *Turing Machine to Compute Binary Carry Sequence*, MCTOC Volume 1 Number 1, November 2015. [Online] Available: http://cs.maryvillecollege.edu/wiki/images/f/f7/MCTOC-v1-n1.pdf . [Accessed: 03 Dec 2015].